

# Supporting the Mathematics Process Goals through Research-based Teaching Practices: Part I

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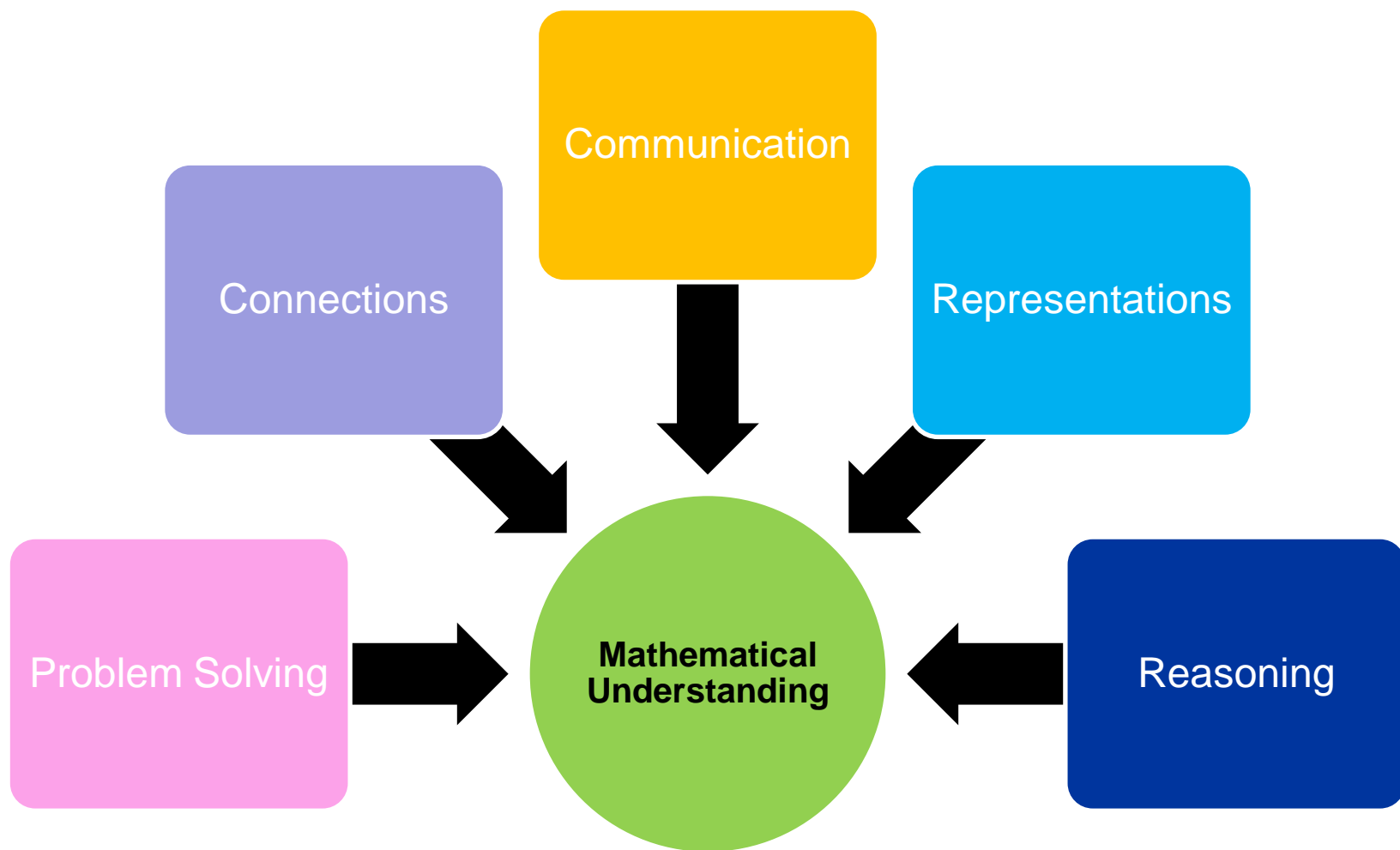
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# Essential Understandings

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- The mathematics process goals (problem solving, communication, reasoning, connections, and representations) play an instrumental role in the teaching and learning of mathematics with understanding.
- There are specific research-based teaching practices that support the implementation of the mathematics process goals.

# Mathematics Process Goals



# Five goals...for students to

- become mathematical **problem solvers** that
- **communicate mathematically**;
- **reason mathematically**;
- **make mathematical connections**; and
- **use mathematical representations** to model and interpret practical situations.

**Process Goals**

## TABLE GROUP DISCUSSION

What teaching practices should be employed to ensure students are engaged in these processes in ways that deepen their mathematical understanding?

**MATHEMATICS THROUGH  
PROBLEM SOLVING**

**VERSUS**

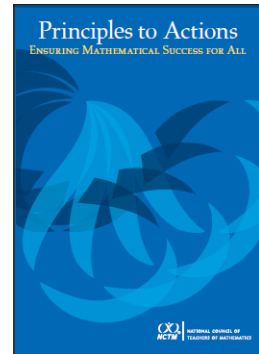
**MATHEMATICS FOR PROBLEM  
SOLVING**

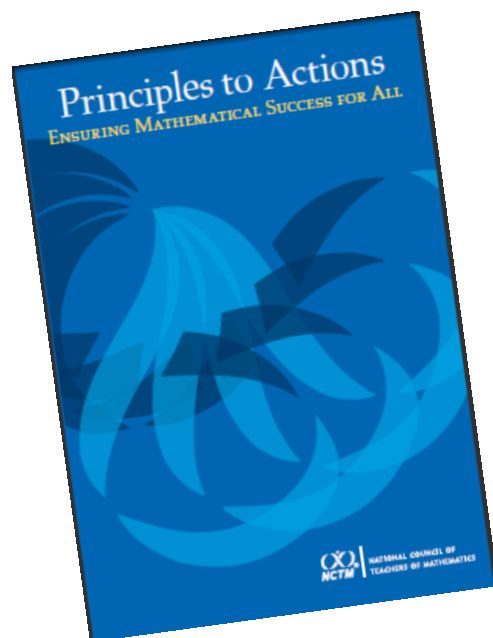
# Mathematics **through** problem solving

- **Interactions** among students and the teacher
- **Communication** of mathematical ideas by students
- Students **analyzing, interpreting, and developing** processes for solving “rich” mathematical tasks
- Teachers **facilitating** learning by **asking strategic questions** and building on ideas that students bring to mathematical tasks
- Teachers **facilitating** students’ **sharing** of ideas, processes, and conclusions

# Principles to Actions: Ensuring Mathematical Success for All

- Describes the **supportive conditions, structures, and policies** required to give all students the power of mathematics
- Focuses on **teaching and learning**
- Engages students in **mathematical thinking**
- How to ensure that mathematics achievement is maximized **for every student**
- Not specific to any standards; **it's universal**





## Mathematics Teaching Practices

**Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

**Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

**Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

**Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

**Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

**Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

**Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

**Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.





# Mathematics Process Goals

## Research-based Mathematics Teaching Practices

The following research-informed framework of teaching and learning reflects the knowledge of mathematics teaching that has accumulated over the last two decades. It represents a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics.

Mathematics Teaching Practices	Process Goal Connections
<i>Establish mathematics goals to focus learning.</i> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	
<i>Implement tasks that promote reasoning and problem solving.</i> Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	
<i>Use and connect mathematical representations.</i> Effective teaching of mathematics engages students in making connections among mathematical representations and understanding the relationships between concepts and procedures and how they are used.	
<i>Facilitate meaningful mathematical discourse.</i> Effective teaching of mathematics facilitates discussion that promotes understanding of mathematics, encourages student approaches and arguments, and builds on student ideas.	
<i>Pose purposeful questions.</i> Effective teaching of mathematics poses purposeful questions to assess student understanding and to promote sense making about important mathematical concepts.	
<i>Build procedural fluency from conceptual understanding.</i> Effective teaching of mathematics builds on students' understanding as a foundation of conceptual understanding so that students can become skillful in using procedures to solve a variety of mathematical problems.	
<i>Support productive struggle.</i> Effective teaching of mathematics encourages students to work individually and collectively, with appropriate supports, to persevere in solving problems and to make sense of productive struggle as they learn mathematical relationships.	
<i>Elicit and use evidence of student understanding.</i> Effective teaching of mathematics uses evidence of student understanding to inform instruction toward mathematical understanding and to assess student learning continually in ways that support learning.	

Process Goal	Notes
<b>Mathematical Problem Solving</b> <ul style="list-style-type: none"> <li>• Build new mathematical knowledge through problem solving.</li> <li>• Solve problems that arise in mathematics and in other contexts.</li> <li>• Apply and adapt a variety of appropriate strategies to solve problems.</li> <li>• Monitor and reflect on the process of mathematical problem solving.</li> </ul>	
<b>Mathematical Communication</b> <ul style="list-style-type: none"> <li>• Organize and consolidate their mathematical thinking through communication.</li> <li>• Communicate their mathematical thinking coherently and clearly to peers, teachers and others.</li> <li>• Analyze and evaluate the mathematical thinking and strategies of others.</li> </ul>	

**AIR-SHARE**

*Use the process goal indicators and align it with the Teaching Practices...*

**Good practices support the process goals.**

*Connections you found with a partner ....*

<ul style="list-style-type: none"> <li>• Use representations to model and interpret physical, social and mathematical phenomena.</li> </ul>	
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## THINK-PAIR-SHARE

*Take a few minutes to look at the process goal indicators and compare it with the Mathematics Teaching Practices...*

## Identify which teaching practices support the individual process goals.

*Share at least two connections you found with a partner ....*

Adapted from: The National Council of Teachers of Mathematics. (p. 9-10, 2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM

- Use representations to model and interpret physical, social and mathematical phenomena.

# Eight High-Leverage Instructional Practices

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- **Use and connect mathematical representations**
- **Facilitate meaningful mathematical discourse**
- Pose purposeful questions
- Build procedural fluency from conceptual understanding
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking



## Mathematical Communication

Students will **use the language of mathematics**, including specialized vocabulary and symbols, to **express mathematical ideas** precisely. **Representing, discussing, reading, writing, and listening** to mathematics will help students to **clarify their thinking** and **deepen their understanding** of the mathematics being studied.

Students who learn to **articulate** and **justify** their own mathematical ideas, **reason** through their own and others' mathematical explanations, and **provide a rationale** for their answers **develop a deep understanding** that is critical to their future success in mathematics and related fields.

The National Council of Teachers of Mathematics. (p. 30, 2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA. NCTM

# Students need opportunities to....

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

National Council of Teachers of Mathematics. (2000).  
*Principles and Standards for School Mathematics*. (p. 206). Reston, VA.

# Taking a Peek inside a Mathematics Classroom

- Use the Mathematical Process Goals for Students Recording Sheet to review the indicators under ‘communication’.

## **Mathematical Communication**

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

- In the note’s section, record specific evidence from the video that the indicators for the process goal of communication are evident in this classroom.

## Table group – round robin

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- What evidence did you find that the process goal, communication, was present in this lesson?
- What evidence suggests that meaningful discourse is a part of this classroom culture?
- Discuss how the purposeful exchange of ideas in the mathematics classroom can create opportunities to elicit and use evidence of student thinking.

# Facilitating Meaningful Mathematical Discourse

## WHAT ARE TEACHERS DOING:

- Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.
- Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.
- Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.
- Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.

## WHAT ARE STUDENTS DOING:

- Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.
- Listening carefully to and critiquing the reasoning of peers, using examples to support or counter-examples to refute arguments.
- Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.
- Identifying how different approaches to solving a task are the same and how they are different.



# Beliefs About Teaching and Learning Mathematics

Beliefs about teaching and learning mathematics	
Unproductive beliefs	Productive beliefs
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.

## REFER to the Beliefs about Teaching and Learning Mathematics Handout

- What productive beliefs are evident in Mrs. Paster's fifth grade classroom?
- What impact do those beliefs have on the classroom discourse?

to ensure that they are not frustrated or confused.

supports productive struggle in learning mathematics.

# Encourage Mathematical Communication with Talk Moves

1. Teacher revoices a student's reasoning for the purpose of clarification and advancing student thinking.
2. Student revoices another student's reasoning to make sense themselves and advance the other students' reasoning and understanding.
3. Asking students to justify or prove someone else's reasoning. Using justification or proof to allow for respectful discussion of ideas.
4. Asking student to build on the group's reasoning by connecting and extending another student's idea.
5. Wait time (means to make the other things happen).

Resource: Anderson, N., Chapin, S. & O'Connor, C. (2011). *Classroom Discussions: Seeing Math Discourse in Action, Grades K-6*. Sausalito: Math Solutions.

With appropriate support and a classroom environment where communication about mathematics is expected, teachers can work to build the capacity of students to think, reason, solve complex problems, and communicate mathematically. This involves creating classroom environments in which intellectual risks and sense-making are expected.

National Council of Teachers of Mathematics. (2000).  
*Principles and Standards for School Mathematics*. (p. 197). Reston, VA.

# Questions???



## Mathematical Problem Solving

Students will **apply** mathematical concepts and skills and the relationships among them **to solve** problem situations of varying complexities. Students also will **recognize and create problems** from real-life data and situations within and outside mathematics and then **apply appropriate strategies** to find acceptable solutions. To accomplish this goal, students will need to **develop a repertoire of skills and strategies** for solving a variety of problem types. A major goal of the mathematics program is to help students become competent mathematical problem solvers.



## Mathematical Reasoning

Students will **recognize reasoning and proof** as fundamental aspects of mathematics. Students will **learn and apply inductive and deductive reasoning skills** to **make, test, and evaluate** mathematical statements and to **justify** steps in mathematical procedures. Students will use logical reasoning to **analyze** an argument and to determine whether conclusions are valid. In addition, students will **learn to apply** proportional and spatial reasoning and to **reason from a variety of representations** such as graphs, tables, and charts.



## Mathematical Connections

Students will **relate concepts and procedures** from different topics in mathematics to one another and see mathematics as an integrated field of study.

Through the application of content and process skills, students will **make connections between different areas of mathematics and between mathematics and other disciplines.**



## Mathematical Representations

Students will **represent and describe mathematical ideas, generalizations, and relationships** with a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should move easily among different representations – **graphical, numerical, algebraic, verbal, and physical** – and recognize that representation is both a *process and a product*.